# Fluency without Equivocation

By Scott Baldridge, Ben McCarty, and Robin Ramos
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For many years a passionate group of math educators has decried the memorization of math facts in grades K-5 as unproductive for learning mathematics. Indeed, the larger education establishment has long known that "blind memorization," i.e., handing students a list of random facts and drilling them with a timed test until the facts are memorized, is no substitute for helping students to memorize their facts through activities that develop their number sense. There can also be negative consequences to giving such "drill and kill" tests. For example, these types of timed tests often give students the wrong impression that, in mathematics, "speed" means "smart." The point: the methods employed to help students memorize and fluently use math facts matter a great deal to their overall understanding and creative use of mathematics in their lives.

Recently, some educators in that passionate group have taken one sentence out of 20,000+ pages of the EngageNY math curriculum and inferred from that sentence that the entire curriculum approaches fluency only through "blind memorization." As the lead writers and mathematicians of the EngageNY curriculum, we feel that the curriculum has been unfairly characterized—that we have been accused of perpetuating the very thing we carefully designed the curriculum to avoid. Therefore we have written this article to reaffirm, *without equivocation,* the following two points for parents, teachers, and other educators involved with the EngageNY curriculum:

- An important goal of the EngageNY curriculum in grades K-5 is for all students to become fluent with the math facts (addition tables, multiplication tables, algorithms, etc.). By fluent we mean students can recall facts without hesitation and can perform routine calculations without thought—similar to speaking a language fluently.

- We intentionally engineered the curriculum to reach this goal through joyful-yet-rigorous activities that develop students' number sense, not through "drill and kill" blind memorization. When it comes to the importance of number sense, we are in complete agreement with the educational establishment as a whole.

In the first part of the article we explain what it means to develop number sense, and why it is important in reaching the goal of helping students become fluent with their facts. The second part of the article shows three examples of activities that we use to develop number sense.

## Developing Number Sense

The sentence that the educators quoted can be found in documents describing the instructional shifts, including the fluency section of the *How To Implement* document for *A Story of Units*, which is a document *about* the PK-5 portion of the EngageNY curriculum. The fluency component

of each lesson is further explained in the *How To Implement* document as having the following purpose:

> "Fluency is designed to promote automaticity by engaging students in practice in ways that get their adrenaline flowing. Automaticity is critical so that students avoid using up too many of their attention resources with lower-level skills when they are addressing higher-level problems. The automaticity prepares students with the computational foundation to enable deep understanding in flexible ways." (page 22)

Flexibility and automaticity *are* key here. A skilled musician, surgeon, athlete, or chef has certain core processes so practiced that they become automatic, thereby freeing up the brain to focus on the larger task, and adapt to the moment. We want students to be able to do the same with mathematics—to take their knowledge of mathematics and use it. For example, a third grader who already knows that $5\times4$ is $20$ is then able to reason that $7\times4$ is just 2 more fours, and therefore $7\times4$ can be broken down into two "easier" parts that they already know from memory:

$$7\times4 = 5\times4 + 2\times4.$$

This is the distributive property in action, which becomes an important concept later in algebra. The student is making use of structure, making use of known facts that can be recalled from memory. Eventually, $7\times4$ becomes a known fact too, so that when encountering a problem like $57\times34$ in a later grade, students are not stumbling over how to multiply $7\times4$ but rather, simply recalling it from memory, able to focus on the process of multiplying two 2-digit numbers.

Indeed, one of the articles the educators cite in their criticism of EngageNY actually *supports* this sensible approach to fluency we took in writing the curriculum. In the research article titled, "Learning by strategies and learning by drill—evidence from an fMRI study," some subjects were trained by blind memorization, while others were trained with a variety of back-up strategies. Both groups were instructed, "to work as fast and accurate as possible." Having tested both groups, the researchers concluded that:

> "Though there is no doubt that skilled and automatic retrieval of arithmetic facts is advantageous in calculation tasks, saving working memory resources, time, and effort, the way to reach this goal should start with back-up strategies providing the understanding of the underlying numerical relations." (Delazer, Ischebeck, Domahs, et al, *NeuroImage*, 2005)

We designed activities in the curriculum that develop skill, flexibility, and automaticity. Good fluency activities engage students in flexible thinking and help them develop their number sense, while pushing them toward the ability to recall key facts from memory. Throwing out the good fluency activities along with the bad and expecting the student to learn math anyway would be like expecting someone to play baseball without developing the ability throw and catch a ball, pick the banjo without developing the skills needed to pick, or conduct surgery before learning to use a scalpel. Certainly bad fluency activities should be eradicated, but not at the expense of the good.

## Examples

Because we basically agree with the passionate group about the difference between bad and good fluency, the negative criticism levied against the EngageNY came as a bit of shock to us. If the educators who made the accusation about the EngageNY curriculum had actually looked at the curriculum materials, they would have seen for themselves that it is brimming over with mental math, counting, and arithmetic activities that develop mathematics with understanding. In what follows, we'll describe three ubiquitous fluency activities from the Engage NY curriculum that exemplify the development of automatic retrieval through the process of understanding underlying numerical relationships:

### The Sprint[1]

At first glance, the Sprint looks quite similar to the timed test many rightly criticize. Both are timed, but the structure and intentional design of a Sprint makes it a completely different experience from "drill and kill" tests.

When administering a Sprint the teacher distributes the first of 2 analogous problem sets (called Sprint A and Sprint B). The students are given 60 seconds to complete as many problems as they can of Sprint A. Next comes a short-but-focused period of time where students analyze the problem set: the Sprint is intentionally structured to encourage students to look for patterns in the problems—patterns that will reappear in Sprint B. For example the following sequence of problems comes from a Sprint in Grade 3, Module 1:

1. 5 + 5 = ___
2. 2 fives = ___
3. 2 + 2 = ___
4. 2 twos = ___

The patterns and relationships from one problem to the next are investigated, articulated by the class as a whole, and used by each student to their advantage in completing Sprint B. Of course, the patterns that students discover are the very number relationships that help build their number sense. Finally, students take and correct Sprint B, and report *how much better they did on Sprint B than on Sprint A.*

Sprints intentionally move from simple to complex problems so that the lowest performing student can always have success with the earlier problems, and the highest performing student is unable to complete all the problems. The goal is not for students to complete a

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[1] Learn more about Yoram Sagher's Sprints at http://ramosgroup.squarespace.com/sprints. Bill Davidson, the author of many of the sprints used in *A Story of Units* has a nice introduction to Sprints available at: http://greatminds.net/maps/math/video-gallery/introduction-to-sprints

set number of problems in a set time (it's not even given a grade), but rather for students to become self-aware of their own improvement.

Thus, students come to learn that they are competing with themselves, which focuses the student on a growth mindset. This alleviates the "speed" anxiety that students often experience with "drill and kill" fluency exercises, but still allows the ticking clock to generate excitement and adrenaline while providing a real way for students to see their own personal growth.

Is memorization one of the long-term goals of Sprints? Absolutely! The Sprints are dealing with ideas that the students will need to use as stepping-stones for understanding later on. But the approach is not blind memorization, but rather the achievement of automaticity through understanding numerical relationships. Having already been introduced to the conceptual underpinnings of the math content by the time the Sprint is given, the Sprint provides an opportunity to practice to automaticity the numerical relationships needed to build the student's number sense.

**Snap**

One of the fluency games that the passionate group of educators suggests is called *Snap It*, where students take a linker cube train with a specified number of cubes. On the signal, they break the train into two parts, hide one part behind their back and then the other children have to work out what the missing part is. We whole-heartedly agree with this activity and include something very much like it in the curriculum. The following application problem comes from Module 4 of Kindergarten:

**Application Problem (5 minutes)**

Materials: (S) 5-stick of linking cubes per student, pencil, paper.

Play a game called Snap with your friend! Show him your 5-stick. Now, put your linking cube stick behind your back. When he says, "Snap!" quickly break your linking stick into two parts. Show him one of the parts. Can he guess the other one? If not, show him. Draw a number bond to show what you did with your cubes. Then, it is his turn! If you have time, play it with a 4-stick, a 3-stick, and a 2-stick!

The game is introduced in an application problem, but is also continued later via fluency activities. Of course, the key skills developed by this game, namely the ability to flexibly decompose numbers, find the missing part, or total, are practiced in numerous other fluency activities as well.

**Skip-Counting**

This activity shows up throughout *A Story of Units* in many different contexts. Students early in elementary school learn to count, then to skip-count by 10, 5 or even 2. Later they learn to skip count by 3. Later still they learn to skip count by unit fractions, or even

measurement units, and include simple conversions. For example students might chorally count together: "1 fourth, 2 fourths, 3 fourths, ONE, 5 fourths, 6 fourths, 7 fourths, TWO!"

Such activities emphasize that even as the unit being counted changes, e.g. counters in Kindergarten, tens in Grade 1, twos and hundreds in Grade 2, fours and fourths in Grade 3, ten thousands in Grade 4, and volume units in Grade 5, we still work with all of these quantities in the exact same ways. These skip-counting activities are also used to help students utilize important mathematics, e.g. the relationship of skip counting to multiplication ("When I skip counted by four to find 3 fours, I got to 12. Three times 4 is 12!"). It also helps students learn the distributive property ("I see that 6 sevens is 42, and 7 sevens is just one more seven, so 7x7=49").[2]

These are just a few of the many types of fluency activities one can find in the EngageNY curriculum. Throughout, all fluency activities are designed to help students see relationships, and thereby develop number sense. Their increased number sense subsequently leads to the development of recall of certain key math facts. Will students recall things at different speeds? Of course. But recall is certainly simpler than always having to rely upon some multistep strategy, and thus will require a lower cognitive load in later grades where it is assumed that students have automaticity.

Let's eliminate blind memorization as a "teaching technique" but let's not eliminate automaticity as one of the goals that good fluency activities can achieve. That's what we aimed for in writing this curriculum: to give teachers sensible activities to do in their classrooms that encourage their students to learn math facts to automaticity, and to add, subtract, multiply, and divide fluently.

Scott Baldridge
Associate Professor of Mathematics
Louisiana State University
Lead Writer and Lead Mathematician, EngageNY Mathematics Curriculum
ScottBaldridge.net (This article and other *Engineering School Mathematics* articles can be found at this website)

Ben McCarty
Assistant Professor Mathematics
University of Memphis
Mathematician, PK-5, EngageNY Mathematics Curriculum
https://umdrive.memphis.edu/bmmccrt1/public/

Robin Ramos
Lead Writer, PK-5, EngageNY Mathematics Curriculum
http://ramosgroup.squarespace.com (this article can be found at this website)

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[2] For a demonstration of this idea check out the following video on *Growing up with Eureka*: http://scottbaldridge.net/2015/01/26/12-21-2014-autumn-multiplying-6x7-7x7-8x7-etc/